

Probability Theory and Introductory Statistics

ALY 6010

Assignment 2

Title: Posterior Probability of HIV test

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**SENSITIVITY:**

Introduction

Sensitivity is defined as the probability of a true positive. True positive is a situation that occurs when both, the test results and the observations are positive. Sensitivity measures the accuracy of determining this true positive.

Analysis

In the case of the HIV testing, sensitivity is the ratio of HIV patients testing positive out of the patients that do possess HIV.

|  |  |  |  |
| --- | --- | --- | --- |
| Test result | Known HIV+ (H+) | Known HIV- (H-) | Total |
| Positive | 990 | 15 | 1005 |
| Negative | 10 | 985 | 995 |
| Total | 1000 | 1000 |  |

Table 1

From this above table, sensitivity is calculated by determining the number of patients that tested positive on the condition that the patients had HIV. Out of the 1000 patients that had HIV, 990 patients were tested to be positive. Therefore,

Sensitivity =990/1000

=**0.99 or 99%**

Conclusion:

A patient having HIV has a 99% likelihood of testing positive after taking the HIV test while 1% of the patients shall test falsely negative.

**SPECIFICITY**

Introduction:

Specificity is defined as the probability of a true negative. True negative is a situation that occurs when both, the test results as well as the observations are negative. Specificity determines the accuracy of determining this true negative.

Analysis

In the case of the HIV testing, specificity is the ratio of the patients that test negative among the patients that do not have HIV.

From the table,

Out of the 1000 patients that do not have HIV, 985 of them test negative.

Mathematically,

Specificity = 985/1000

= **0.985 or 98.5%**

Conclusion:

A patient that does not have HIV has a 98.5% likelihood of testing negative and a 1.5% probability of testing falsely positive.

**INTERPRETATION OF POSITIVE TEST RESULT**

Introduction:

A patient undergoes a test to determine whether she has HIV The HIV test has a positive outcome, meaning that the test identifies her as a carrier of the virus.

Analysis:

In this situation, we know that the patient has tested positive and thus, need to determine the probability of having HIV.

This can be calculated by determining the ratio of number of patients that test positive and have the virus against the total number of patients that are tested as positive.

Mathematically,

P(H+|T+) = Probability of having the virus while testing positive

=990/ (990+15)….. (From Table 1)

=**0.98507 or 98.507%**

Conclusion:  
There is a 98.507% probability that the patient is carrying the virus if her test result has come back positive.

**HIV PREVALENCE IN DIFFERENT REGIONS OF THE WORLD:**

|  |  |
| --- | --- |
| Region | Adult prevalence |
| Sub-Saharan Africa | 0.059 |
| South and South East Asia | 0.006 |
| East Asia | 0.001 |
| Latin America | 0.005 |
| North America | 0.008 |
| Western and Central Europe | 0.003 |
| Eastern Europe and Central Asia | 0.009 |
| Middle East and North Africa | 0.002 |
| Caribbean | 0.012 |
| Oceania | 0.004 |

Table 2

The probability of testing positive is the summation of probabilities testing positive while having HIV as well as testing positive while not having HIV.

Mathematically,

P(T+)= P(T+|H+)\*P(H+) + P(T+|H-)\*P(H-)

Where,

*P(T+)= Probability of testing positive*

*P(T+|H+)= Probability of testing positive while having HIV [0.99]*

*P(T+|H-)= Probability of testing positive while not having HIV [0.015]*

*P(H+)= Probability of having HIV*

*P(H-)= Probability of not having HIV [or 1- P(H+)]*

In the case of Sub-Saharan Africa,

P(H+)=0.059 and P(H-)=0.941

Now,

P(T+)= 0.99\*0.059 + 0.015\*0.941

Therefore, **P(T+)= 0.072525** (for Sub-Saharan Africa).

Similarly, in the case of East Asia:

**P(T+)= 0.015975** (probability of testing positive for patients from East Asia)

Calculation of the posterior probability is done with the formula, given by Bayes’ theorem:

**CASE I (EAST ASIA)**

A patient from East Asia undergoes the test and the test result comes back positive. If the region and the associated prevalence were unknown, she would have to be worried as the accuracy of results in such a scenario has a probability of 98.507%.

Since, the region and the prevalence of disease for the same is known, posterior probability can be used to increase the reliability and accuracy of the result.

The posterior probability of actually carrying the virus, on the condition of testing positive is given by:

P(H+|T+)= (P(T+|H+)\* P(H+))/P(T+)

= 0.99\*0.001/0.015975

= **0.06197 (or 6.197%)**

This is very surprising because while the test results alone indicated that the probability of having HIV is 98.507%, the posterior probability calculated shows that the actual probability is only 6.197%- a very low chance of having the disease after taking the probability of prevalence into consideration.

**CASE II (SUB-SAHARAN AFRICA)**

If a patient has taken the test and the result has come back as positive and if that patient is from Sub-Saharan Africa, then the posterior probability of carrying the virus can be calculated using:

P(H+|T+)= (P(T+|H+)\* P(H+))/P(T+)

= 0.99\*0.059/0.072525

=**0.80537 (or 80.537%)**

The patient should be worried because there is a high probability that the test results were accurate- meaning that she has HIV, leaving less than 20% room for inaccuracy.

A patient from Sub-Saharan Africa should be more worried than the patients from East Asia as they run a higher risk of having HIV if the test results are positive.

**SECOND TRIAL:**

If the patient from East Asia has tested positive and goes to another clinic for diagnosis and gets a second, independent HIV test and this test turns out to be positive too, then the posterior probability of having HIV is given by the formula, according to Bayes’ theorem:

P(H+|T+∩T++):

*P(T+⋂T++│H+)= Probability of testing positive in both the tests while having HIV*

*P(T+⋂T++│H-)= Probability of testing positive in both tests while not having HIV*

Since the second test is independent of the first test, the probability of obtaining similar outcome remain the same:

*P(T++|H+)= P(T+|H+) & P(T++|H-)=P(T+|H-)*

*P(T++|H+)= Probability of testing positive in the second test while having HIV*

*P(T++|H-)=Probability of testing positive in the second test while not having HIV*

Now, P(H+|T+∩T++)

Also, *P(T+∩T++|H+)= P(T+|H+)\*P(T++|H+)*

*P(T+∩T++|H-)=P(T+|H-)\*P(T++|H-)*

=0.81344 (or 81.344%)

The patient from East Asia should be worried that she has HIV since the likelihood has increased exponentially from 6.197% to 81.344% after taking the second test and obtaining a positive result. It has added more value and significance to the outcome, obtained at the end of the first test.

**Reference:**

C.R. Bhatti, J.L. Wightman. Conditional Probability and HIV Testing: A Real-World Example. The American Statistician. Vol. 62, No. 3 (Aug. 2008)